ANALYTICAL GEOMETRY IN THREE DIMENSIONS

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IN THIS SECTION, YOU WILL:

- PLOT A POINT IN 3-DIMENSIONS.
- CALCULATE 3-DIMENSIONAL DISTANCE AND MIDPOINT.
- FIND AND GRAPH THE EQUATION OF A SPHERE.
- FIND A TRACE OF A SPHERE.

- Points in 3 dimensions
 - (x, y, z)
 - *x* comes out/into of paper
 - *y* is left/right
 - z is up/down
- Graph by moving out the *x*, over the *y*, then up the *z*.
 - Graph A(5, 6, 3)
 - Graph B(-2, -4, 0)

- Distance Formula
 - In 2-D:
 - In 3-D: (just add the z)

- Midpoint Formula
 - In 2-D:
 - In 3-D: (just add the z)

- Equation of Circle (2-D)
- Equation of Sphere (3-D) (just add z)
 - Center is (h, k, j), r = radius
 - Graph by plotting the center and moving each direction the radius
 - Graph



- Trace (like intercepts for a sphere)
 - Draw the *xy* trace for



IN THIS SECTION, YOU WILL:

- USE VECTOR OPERATIONS IN THREE DIMENSIONS.
- FIND THE ANGLE BETWEEN VECTORS.

- Vectors in 2-D
- Vectors in 3-D (just add z)
- To find a vector from the initial point to the terminal point

- If and,
- Addition
 - Add corresponding elements
- Scalar multiplication
 - Distribute



- If and,
- Dot Product
- Magnitude
- Unit vector in the direction of



• Angle between vectors

- If (and)
 - Then vectors are orthogonal
- If
 - Then vectors are parallel



- Let and
- Find

• Find unit vector in direction of

• Find

- Let and
- Find

• Find the angle between and

• Are and parallel, orthogonal, or neither?

• Parallel if

- Are , , and collinear?

IN THIS SECTION, YOU WILL:

- EVALUATE A CROSS PRODUCT.
- USE A CROSS PRODUCT TO SOLVE AREA AND VOLUME PROBLEMS.



- is unit vector in x, is unit vector in y, and is unit vector in z
- and
- If and , find



• Properties of Cross Products

• is orthogonal to and

• If, then and are parallel



- Area of a Parallelogram
 - where and represent adjacent sides

• Triple Scalar Product (shortcut)

- Volume of Parallelepiped
 - (3-D parallelogram)
 - where , , and represent adjacent edges



IN THIS SECTION, YOU WILL:

- WRITE AN EQUATION FOR A LINE IN THREE DIMENSIONS.
- WRITE AN EQUATION FOR A PLANE.
- FIND THE ANGLE BETWEEN TWO PLANES.
- GRAPH A PLANE.

• Lines

- Line *L* goes through points *P* and *Q*
- is a direction vector for *L*
- Start at *P* and move any distance in direction to get some point *Q*
- because they are parallel
 - General form



- Parametric Equations of Line
 - Take each component of the general form and solve for *x*, *y*, or *z*.
 - We used these when we solved 3-D systems of equations and got many solutions

- Symmetric Equation of Line
 - Solve each equation in parametric equations for *t*

• Find a set of parametric equations of the line that passes through (1, 3, -2) and (4, 0, 1).

- Planes
- because they are perpendicular
- Standard form

• General form



• Find the general equation of plane passing through , , and



• Angle between two planes

- Find the angle between normal vectors
- Normal vectors are coefficients in the equations of the plane



• Distance between a Point and a Plane



- Graphing planes in space
 - Find the intercepts
 - Plot the intercepts
 - Draw a triangle to represent the plane
- Sketch

