1. ABERRATIONS

1.1 Introduction

Aberrations (or) defects of a lens:

The deviations from the actual size, shape and position of an image as calculated by simple equations are called aberrations produced by a lens.

Types of aberrations:

Aberrations are mainly classified in the two types

(1) Monochromatic aberration.

(2) Chromatic aberration.

1.2 Monochromatic aberration

Mono chromatic aberrations are further classified in to five types, they are,

(i) Spherical aberration

(ii) Comatic aberration (or) coma

(iii) Astigmatism

(iv) Distortion

(v) Curvature

1.3 Spherical Aberration:

The inability of the lens to form a point image of an axial object is called "spherical aberration".

Explanation:

Consider a point source "O" of monochromatic light placed on the axis of a convex lens as shown in the figure (1).

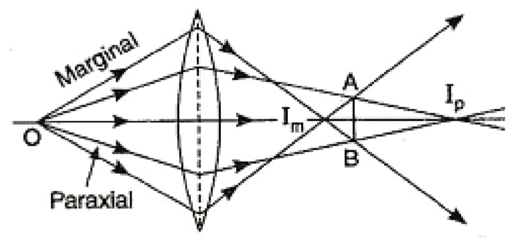

Fig. (1)

The rays which are incident near the axis are called **"paraxial rays"**. They come to focus at a point I_P. The rays incident near the rim of the lens are called **"marginal rays"**. They come to focus at a point I_m. The intermediate rays are brought to focus between I_m and I_P.

From the figure, it is clear that the paraxial rays form the image at a point farther than the marginal rays. Thus, the image is not sharp at any point on the axis. This defect of the lens is called spherical aberration.

1.3.1. Reasons for spherical aberration:

1. The lens can be supposed to be divided in to number of circular zones. As the focal length slightly varies with the radius of the zone, different zones have different focal lengths.
2. The focal length of marginal zone is lesser than the focal length of paraxial zone; hence marginal rays are focused first.
3. The marginal rays suffer greater deviation than the paraxial rays because, they are incident at a greater height than the paraxial rays.

1.3.2. Types of spherical aberration:-

Spherical aberrations are classified into two types.

(i) Longitudinal spherical aberration

(ii) Lateral spherical aberration.

1.3.3. Longitudinal Spherical Aberration:

Consider a point source "O" of monochromatic light placed on the axis of a convex lens as shown in the figure (2).

From the figure (2) it is clear that, the paraxial rays of light form the image I_P at a longer distance from the lens than the image I_m formed by marginal rays. The image is not sharp at any point on the axis.

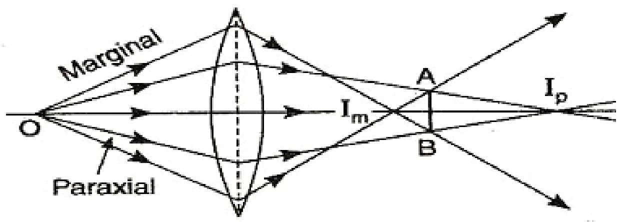

Fig. (2)

The distance between I_m and I_P represents the **"longitudinal or axial spherical aberration."**

1.3.4. Lateral spherical aberration:

Consider a point source "O" of monochromatic light placed on the axis of convex lens as shown in the figure (3).

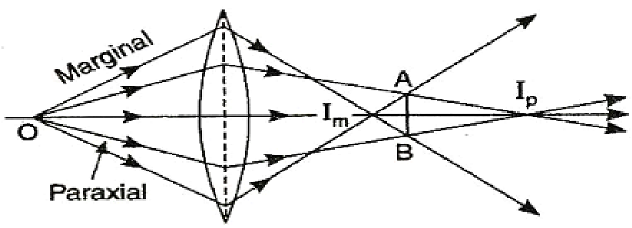

Fig. (3)

The paraxial rays forms the image I_P at longer distance from the lens that the image I_m formed by marginal rays.

If a screen is placed normal to the principle axis at I_m, then the image formed on the screen consists of a circular disc. A disc image would again be obtained if the screen is placed at I_P.

If the screen is moved in between I_m and I_P then the size of the disc is minimum in the position AB where the paraxial and marginal rays cross each other. The disc AB of minimum diameter is called "circle of least confusion". The position of AB corresponds to the position of the best image.

The radius of circle of least confusion is called the **"lateral spherical aberration."**

Note: The spherical aberration produced by a convex lens is positive.

The spherical aberration produced by a concave lens is negative.

1.3.5. Minimization of Spherical aberration:

Spherical aberration can be minimized by the following methods

i) By using stops

ii) By the use of plano-convex lenses – by dividing the deviation equally

iii) By using plano-convex lenses separated by distance

iv) By using two suitable lenses in contact

(i) By using stops:

The spherical aberration can be minimized by using stops. The stops used are in such a way that they permits axial rays and stops the marginal rays (or) the stops permits marginal rays and stops paraxial rays. By using stops, the intensity of final image reduces hence; this method is not generally used.

(ii) By the use of plano-convex lenses (or) by dividing the deviation equally:-

The spherical aberration can be minimized by using plano–convex lenses. The spherical aberration is proportional to the square of the total deviation produced by the lens.

If δ is the total deviation produced by the lens. δ_1 and δ_2 are the deviations produced at the two surfaces of the lens respectively then,

Spherical aberration $\quad \alpha \ \delta^2$

$$\alpha \ (\delta_1 + \delta_2)^2$$

$$\alpha \ (\delta_1 - \delta_2)^2 + 4\ \delta_1, \delta_2$$

From the above expression it is clear that, the spherical aberration is minimum if $\delta_1 = \delta_2$. That is, the deviation is equally divided at the two surfaces.

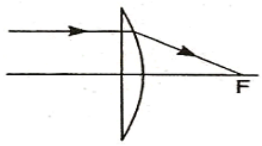

Fig. (4)

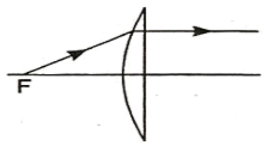

Fig. (5)

If a ray parallel to the principle axis is incident on the plane surface of a plano-convex lens, then there will be no deviation at plane surface and the whole deviation will be on the second surface that is, convex surface as shown in figure (4). When the abject lies at focus towards curved surface then a parallel beam of light passes through the lens hence, there will be no deviation as shown in figure (5).

(iii) By using two plano – convex lenses separated by a distance:

The spherical aberration can be minimized by using two plano convex lenses of the same material separated by a distance equal to the difference of their focal lengths. In this case the deviation produced in a light ray is spread over four surfaces and is shared equally by the two lenses.

Explanation:

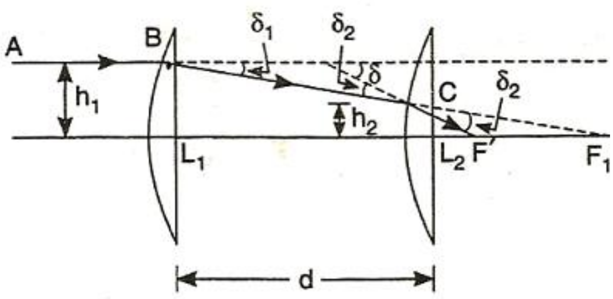

Fig. (6)

Let L_1 and L_2 are the two plano-convex lenses of focal lengths f_1 and f_2 separated by a distance "d" as shown in the figure (6).

Let a ray of light AB coming parallel to the axis meet the lens L_1 at a height h_1 and suffers a deviation $\delta_1 = \dfrac{h_1}{f_1}$. This is directed towards F_1

The refracted ray BC strikes the lens L_2 at height h_2 and suffers a deviation $\delta_2 = \dfrac{h_2}{f_2}$.

The emergent ray meets the axis at F_2.

For minimum spherical aberration,

$$\delta_1 = \delta_2$$

$$\Rightarrow \frac{h_1}{f_1} = \frac{h_2}{f_2} \quad \Rightarrow \frac{h_1}{h_2} = \frac{f_1}{f_2} \qquad \ldots(1)$$

From similar triangles BL_1F_1 and CL_2F_1

We get, $$\frac{BL_1}{CL_2} = \frac{L_1F_1}{L_2F_1}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{L_1F_1}{L_1F_1 - L_1L_2} \Rightarrow \frac{h_1}{h_2} = \frac{f_1}{f_1 - d} \quad \ldots (2)$$

From equations (1) and (2) we get,

$$\frac{h_1}{h_2} = \frac{f_1}{f_2} = \frac{f_1}{f_1 - d} \Rightarrow f_1 - d = f_2$$

$$\Rightarrow \mathbf{d = f_i - f_2}$$

Thus, in order to minimize the spherical aberration, the distance between the lenses must be equal to the difference of their focal lengths.

Note: The general rule to minimize the spherical aberration in a plano convex lens is that the convex side should face the incident or emergent beam whichever is more parallel to the axis.

(iv) By using two suitable lenses in contact:

Spherical aberration for a convex lens is positive and that for a concave lens is negative. By using a suitable combination of convex and concave lenses, spherical aberration can be minimized. The difficulty with this combination is that it works only for a particular pair of object and image for which it is designed.

1.4. Comatic aberration (or) Coma:

When a point object is situated slightly off the principle axis of the lens, then the image of the point object formed by the lens is found to have an "egg – like" or "comet – like" shape. This defect is called "comatic aberration" or "Coma".

Explanation:

Consider an off axial point A in the object OA placed on the axis of the convex lens as shown in the figure (7).

Let the lens be divided into different zones. Consider the image of point A formed by the different zones of the lens. The rays leaving point A and passing through different zones of lens as (1,1), (2,2); (3,3) etc are brought to focus at different points I_1, I_2, I_3 etc,

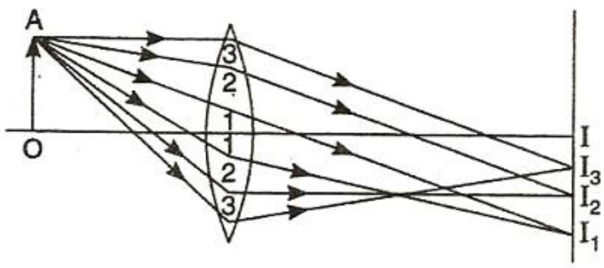

Fig. (7)

So, the image is not in a perfect focus. This defect is called as **"coma".**

1.4.1. Reasons for Coma:

1. Different zones of the lens produce different lateral magnifications. Due to this reason, the rays starting from point A and passes through different zones of the lens are focused at different points.
2. Each zone forms the image of a point in the form of a circle called "Comatic Circle". Consider a particular zone of the lens as shown in figure (8). Let the zone has pairs of diametrically opposite points like (a,a); (b,b); (c,c); (d,d). The rays starting from point A and passing through these pairs will be focused at a^1, b^1, c^1 and d^1 respectively as shown in figure (9). The points a^1, b^1, c^1 and d^1 lie on a circle called as **"Comatic Circle".**

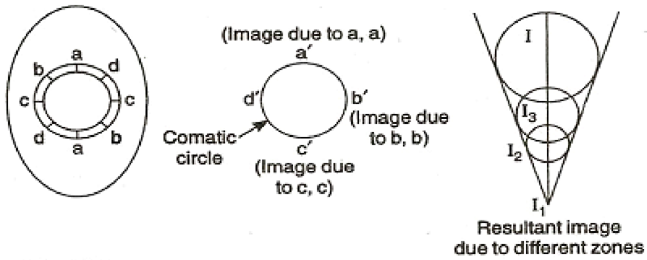

Fig. (8) Fig. (9) Fig. (10)

The radius of comatic circle increases as the radius of the zone of the lens increases. Thus at I_1 the comatic circle is a point. Above I_1, the circle of increasing radii is obtained. The shapes of these circles are as shown in figure (10). The resultant image is in "comet shape".

1.4.2. Minimization of Coma:

1. Coma may be reduced to a certain extent by the use of proper stop placed at a suitable distance from the lens. This stops the outer zones and allows only the central zones to refract the rays.
2. Coma may be minimized by designing lenses of suitable shapes and materials.
3. Abbe, a German optician showed that, coma can be eliminated if a lens satisfies the Abbe's sine condition,

$$\mu_1 y_1 \sin\theta_1 = \mu_2 y_2 \sin\theta_2$$

Here, θ_1 and θ_2 are the angles made by the conjugate rays with the axis, μ_1 and μ_2 are the refractive indices of the object and image spaces, y_1 and y_2 are the lengths of object and image for a particular zone.

The lateral magnification of the image is given by,

$$\frac{y_2}{y_1} = \frac{\mu_1 \sin\theta_1}{\mu_2 \sin\theta_2}$$

When the medium on both sides of the lens is same then, the lateral magnification is ,

$$\frac{y_2}{y_1} = \frac{\sin\theta_1}{\sin\theta_2} \qquad (\because \mu_1 = \mu_2)$$

If the lateral magnification is same for all the rays of light irrespective of the angles θ_1 and θ_2 then coma may be eliminated. Thus, Coma can be eliminated if $\frac{\sin\theta_1}{\sin\theta_2}$ = constant.

A lens satisfying the condition, $\frac{\sin\theta_1}{\sin\theta_2}$ = constant is called as an **"aplanatic lens".**

1.5. Astigmatism:

When a point object "O" is situated far off from the axis of a lens, then the image formed by the lens is not in a perfect focus. The image consists of two mutually perpendicular lines separated by a finite distance. The two lines lie in perpendicular planes. This defect of the image is called as **"astigmatism".**

Explanation:

Let a point object "O" is situated far off from the axis of a lens as shown in the figure (11).

A plane M_1 M_2 containing the principle axis of the lens is called "meridian plane". A plane S_1 S_2 perpendicular to meridian plane and passing through the principle axis of the lens is called "sagittal plane" of the lens.

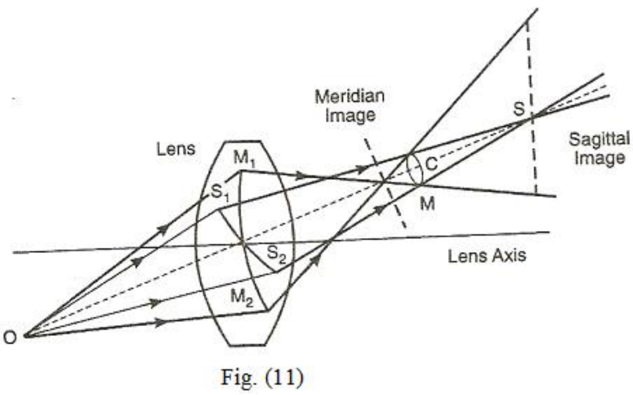

Fig. (11)

Consider the rays from the object "O" passing through the two planes M_1M_2 and S_1S_2. The rays through the meridian plane after refraction come to a horizontal line focus "M", and the rays through the sagittal plane come to a focus farther away from the lens in a vertical line focus "S".

If a screen is moved between M and S, an irregular patch of light is obtained. Thus the image never approaches to a point image. This defect is called as "astigmatism".

When the screen is moved between S and M then, the patch of light reduces to circle "C" as shown in the figure (11). This circle is called as "circle of least confusion". This circle is the nearest approach to a point image.

The difference between the lines M and S is a measure of astigmatism and is called the **"astigmatic difference".**

1.5.1. Minimization of astigmatism:

i) By using stops

ii) By using combination of lenses

i) By using stops

Astigmatism can be minimized with the help of stops by placing them is suitable position as shown in figure (12).

Due to this the rays making large angles with the axis are cut off and the defect is eliminated. Only oblique rays are permitted to form the image.

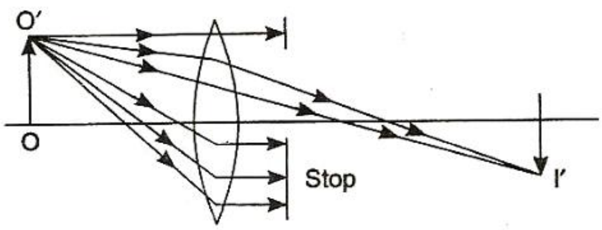

Fig. (12)

ii) By using combination of lenses:

In case of convex lens the astigmatic difference is positive. In case of concave lens the astigmatism difference is negative.

Therefore, a suitable combination of convex and concave lenses separated by a suitable distance may be used to reduce the astigmatism. Such a combination is called as an "anastigmat". This is used in cameras.

1.6. Curvature of the field:

The image of an extended plane object due to a single lens is not flat but will be a curved surface. The central portion of the image nearer the axis is in focus but the outer region of the image away from the axis are blurred. This defect is called the curvature of the field.

Explanation:

Curvature of the field arises due to the fact that the focal length of paraxial rays is greater than the focal length of marginal rays.

The curvature of the field in the image formed by a convex lens is as shown in figure (13) and figure (14). The real image AB formed by a convex lens curves towards the lens as shown in figure (13). The virtual image A^1B^1 formed by a convex lens curves away from the convex lens as shown in figure (14).

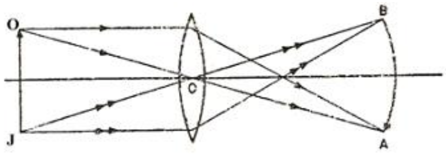

Fig. (13)

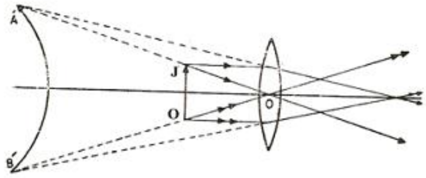

Fig. (14)

The curvature of the field present in the image formed by a concave lens is as shown in figure (15).

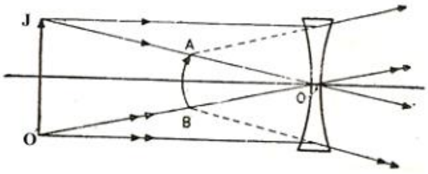

Fig. (15)

1.6.1. Elimination of Curvature:

1. In case of thin lens, curvature can be eliminated by placing a stop in a suitable position in front of the lens.
2. Curvature of the field may be eliminated by using meniscus shaped lenses in combination with suitable stops.
3. The radius of curvature R of the final image for a system of thin lenses is given

by, $\frac{1}{R} = \sum \frac{1}{\mu_n f_n}$ where μ_n = Refractive index of n^{th} lens, f_n = focal length of n^{th} lens.

For flat image R = ∞ $\quad \therefore \frac{1}{R} = \sum \frac{1}{\mu_n f_n} = \frac{1}{\infty} = 0$

For two lenses placed in air, $\frac{1}{\mu_1 f_1} + \frac{1}{\mu_2 f_2} = 0 \quad \Rightarrow \mu_1 f_1 + \mu_2 f_2 = 0$

This is called **"Petzwal's condition"** for the elimination of curvature.

This condition holds good either the lenses are in contact or separated by a distance.

1.7. Distortion:

The image of a square object formed by a lens is not a square. This defect is called as distortion.

This defect arises due to the fact that, the magnification produced by the lens for different parts of the object are different, because the different parts of the object are having different axial distances from the lens. Hence different parts of the object are magnified differently.

1.7.1. Types of distortion

Distortion is of two types

(i) Barrel shaped distortion

(ii) Pin cushion distortion

(i) Barrel shaped distortion:

Consider a rectangular object as shown in figure (16). The convex lens forms a barrel shaped image due to barrel shaped distortion as shown in figure (17).Barrel shaped distortion arises due to the fact that, the magnification decreases with the increase of axial distance from the centre of the lens.

Fig. (16)

Fig. (17)

(ii) Pin-cushion shaped distortion:

If a convex lens forms a pin cushion shaped image of a rectangular object as shown in figure (18), than the distortion is called as pin-cushion shaped distortion.

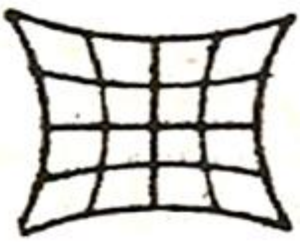

Fig. (18)

Pin - cushion shaped distortion arises due to the fact that, the magnification increases with the increase of axial distance from the centre of the lens.

1.7.2. Minimization of distortion:

(1) Distortion can be minimized by using thin lenses

(2) Distortion can be minimized by using suitable stops.

To eliminate distortion, a stop is placed in between two symmetrical lenses as shown in figure (19), so that the pin-cushion distortion produced by the first lens is compensated by the barrel - shaped distortion produced by the second lens. This arrangement is called **"orthoscopic doublet"** or a **"rapid rectilinear lens"**.

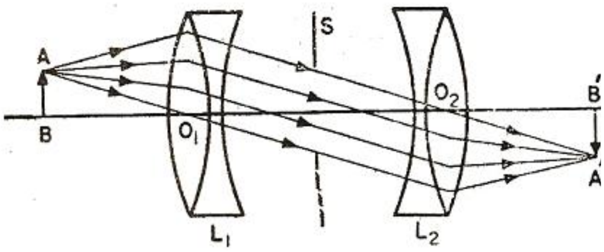

Fig. (19)

1.8 Chromatic aberration:

If the light is not monochromatic then the image formed by the lens becomes multicolored and the defect is called as "Chromatic aberration".

Explanation:

The chromatic aberration occurs due to the fact that refractive index μ varies with colour.

When a parallel beam of white light is refracted through a lens, violet rays meet fist and the red rays meet at farthest point from the lens as shown in the figure (20). This is because, the refractive index of violet colour (μ_v) is greater than refractive index of red colour (μ_R) and hence the focal length of red colour (f_R) is greater than the focal length of violet colour (f_v).

i.e., as $\mu_v > \mu_R$; $f_R > f_V$ because, $\mu \;\alpha \frac{1}{f}$.

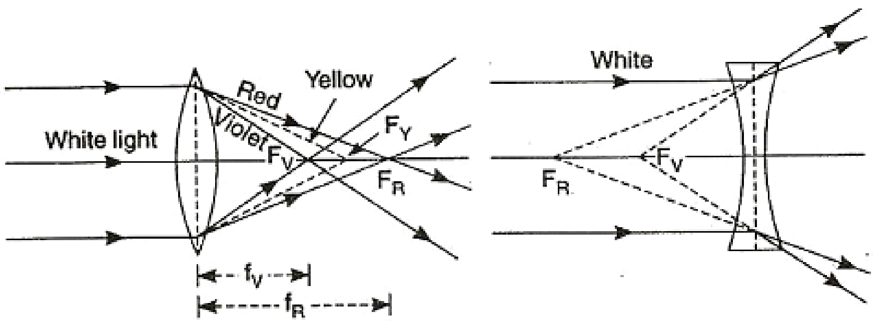

Fig. (20)

If a screen is placed at F_V then the centre of the image would be violet while the outer surface will be red. If a screen is placed at F_R then the centre of the image would be red while the outer surface will be violet. For all positions between F_V and F_R the image is blurred. Thus the image of a white object formed by a lens is coloured and blurred. This defect of the image is known as **"chromatic aberration".**

1.8.1. Types of chromatic aberration:-

Chromatic aberration is mainly two types

i) Longitudinal chromatic aberration

ii) Lateral chromatic aberration

1.8.2. Longitudinal Chromatic Aberration:

The formation of images of different colours in different positions along the axis of the lens is known as longitudinal (or) axial chromatic aberration.

Explanation:- Let a white point object "O" is placed on the axis of a lens as shown in the figure (21).

The violet and red images of the object are formed on the axis of the lens at I_V and I_R respectively. The distance between I_V and I_R represents **longitudinal chromatic aberration**.

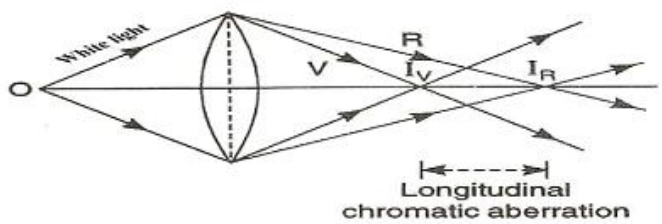

Fig. (21)

$\therefore$ Longitudinal chromatic aberration = $I_R - I_V$.

Note (i) : If the object is at infinity, then the longitudinal chromatic aberration is equal to the difference in the focal lengths of red and blue colours.

Note (ii) :- In case of convex lens longitudinal chromatic aberration is positive and in case of concave lens it is negative.

1.8.3. To Derive the relation for longitudinal chromatic aberration of a thin lens when the object is situated at infinity:-

The relation for focal length f of a lens is, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Where, μ=refractive index of the material of the lens

R_1 and R_2 are the radii of curvatures of two curved surfaces

The relations for fv, fr and fy can be written as

$$\frac{1}{f_v}=(\mu_v-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \ldots(1)$$

$$\frac{1}{f_R}=(\mu_R-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \ldots(2)$$

$$\frac{1}{f_y}=(\mu_y-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \ldots(3)$$

Where fv, f_R and f_y are focal lengths of the lens for violet, red and yellow colours respectively. μ_v, μ_R and μ_y are the refractive indices for violet, red and yellow colours respectively.

Subtracting equation (2) from equation (1) we get.

$$\frac{1}{f_v}-\frac{1}{f_R}=(\mu_v-\mu_R)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \Rightarrow \frac{f_R-f_v}{f_R f_v}=\frac{(\mu_v-\mu_R)\,(\mu_y-1)}{(\mu_y-1)}\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \ldots(4)$$

From equations (3) and (4) we get, $\frac{f_R-f_v}{f_v f_R}=\frac{(\mu_v-\mu_R)}{(\mu_y-1)}\times\frac{1}{f_y}$

$$\Rightarrow \frac{f_R-f_v}{f_y^2}=\frac{\mu_v-\mu_R}{\mu_y-1}\times\frac{1}{f_y}$$

$$(\therefore f_v f_R = f_y^2 = f = \text{average focal length of } f_v \text{ and } f_R)$$

$$\Rightarrow \frac{f_R-f_v}{f_y}=\omega \qquad \left(\because \frac{\mu_v-\mu_R}{\mu_y-1}=\omega\right)$$

$\Rightarrow$ $f_R - f_v = \omega f_y$ = longitudinal chromatic aberration

$$\mathbf{f_R - f_v = \omega f} \qquad \text{where } f=\frac{f_R+f_v}{2}$$

Where, ω = dispersive power of the lens

That is, the longitudinal chromatic aberration for a thin lens for an object situated at infinity is equal to the product of mean focal length and dispersive power of the lens.

1.8.4. To Derive the relation for longitudinal chromatic aberration of a thin lens when the object is situated at finite distance:-

For a thin lens, the relation between object distance "u", image distance "v" and focal length "f" is,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad \ldots(1)$$

For an object placed at fixed finite distance “u”, the image distance v and focal length f changes with colour.

Differentiating equation (1), we get,

$$-\frac{dv}{v^2} = -\frac{df}{f_2} \qquad \ldots(2) \quad (\because u = \text{constant})$$

Let the image distances for red and violet colours be v_r, v_v respectively and the focal lengths for red and violet colours be f_R and f_v respectively. Then,

$$dv = v_R - v_v$$
$$df = f_R - f_v$$

We know that, $f_R - f_v = \omega\, f_y$

When, ω = dispersive power of the length, f_y = focal length of yellow colour.

$$\therefore df = f_R - f_v = w\, f_y$$

Substituting the values of dv and df in equation (2) we get,

$$\frac{v_R - v_v}{v_y^2} = \frac{\omega f_y}{f_y^2} \qquad (\because v = v_y, f = f_y)$$

$$\frac{v_R - v_v}{v_y^2} = \frac{\omega v_y^2}{f_y} \Rightarrow \mathbf{\frac{v_R - v_v}{v} = \frac{\omega\, v}{f}} \qquad \ldots(3) \quad \left(\because v = \frac{v_R + v_v}{2}, \quad f = \frac{f_R + f_v}{2}\right)$$

The above equation represents longitudinal chromatic aberration for an object situated at finite distance.

From equation (3) the following points may be considered

The longitudinal chromatic aberration depends on,

(i) The distance of the material of the lens

(ii) The mean focal length ‘f’.

(iii) The image distance ‘v’, for mean ray, which depends on the object distance ‘u’.

1.8.5. Lateral Chromatic Aberration:

When the images of different colours are formed by the lens of different size then the defect is called as lateral chromatic aberration.

This is due to the fact that the magnification of white object is different for different colurs.

Explanation:

Let an object AB be placed infront of a convex lens as shown in the figure (22).

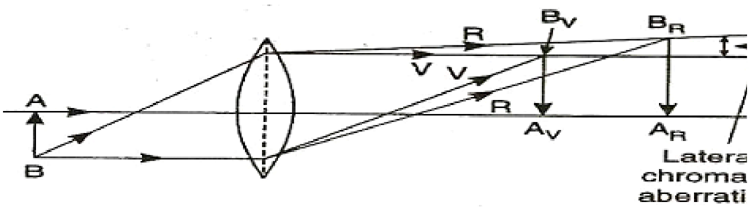

Fig. (22)

The lens forms the image of white object AB as B_V A_V and B_R A_R respectively in violent and red coloures. The images of other colours lie in between the two.

From the figure it is clear that the image of red colour is larger than the image of violet colour. The difference (B_R A_R - B_V A_V) in the sizes is a measure of lateral chromatic aberration.

1.8.6. Achromatism of lenses (or) Achromatic doublet:

The minimisation or removal of chromatic aberration is called as **"achromatism".**

When two lenses are placed in such a way that the image formed by them is free from chromatic aberration then the combination of lenses is called as "achromatic doublet".

The conditions of achromatism can be obtained in two cases.

i) Achromatism for two lenses in contact

ii) Achromatism for two lenses separated by a distance

1.8.7. Achromatism for two lenses in contact:

An achromatic doublet is formed in such a way that all colours come to focus at one point.

To have an achromatic combination of two lenses un contact, one of the lenses should be convex made of crown glass and the other should be concave made of flint glass.

Explanation:

Let f_1 and f_2 be the focal lengths of the two lenses in contact. ω_1 and ω_2 be their dispersive powers between the two wavelengths for which the combination is to be achromatic (dispersive power $\omega = \frac{d\mu}{\mu - 1}$)

If F is the combined focal length of the lenses in contact then, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

Differentiating the above equation, we get

$$d\left(\frac{1}{F}\right) = d\left(\frac{1}{f_1}\right) + d\left(\frac{1}{f_2}\right)$$

$$\Rightarrow d\left(\frac{1}{F}\right) = \frac{\omega}{f_1} + \frac{\omega}{f_2} \qquad \ldots(1)$$

$$\because d\left(\frac{1}{f_1}\right) = \frac{\omega_1}{f_1} \text{ and} \because d\left(\frac{1}{f_2}\right) = \frac{\omega_1}{f_2}$$

For an achromatic combination, $\frac{1}{F}$ should not change with colour.

That is , $\Rightarrow d\left(\frac{1}{F}\right) = 0$ …(2)

From equations (1) and (2) we get,

$$\frac{\boldsymbol{\omega_1}}{\mathbf{f_1}} + \frac{\boldsymbol{\omega_2}}{\mathbf{f_2}} = \mathbf{0}$$

This is the required condition for two lenses in contact to acquire achromatism.

The ratio of focal lengths of the two lenses in an achromatic doublet is given by,

$$\frac{\mathbf{f_1}}{\mathbf{f_2}} = \frac{\mathbf{-\boldsymbol{\omega_1}}}{\boldsymbol{\omega_2}}$$

As ω_1 and ω_2 are positive quantities, one of f_1 and f_2 must be negative that is, if one lens is convex then the other should be concave as shown in the figure (23).

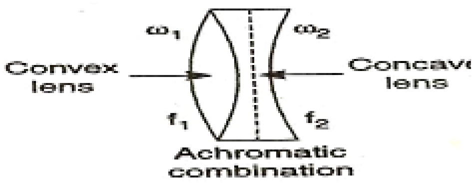

Fig. (23)

1.8.8. Achromatism for two lenses separated by a distance:

Consider two lenses of focal lengths f_1 and f_2 separated by a suitable distance x as shown in the figure (24). Let ω_1 and ω_2 are the dispersive powers of the two lenses.

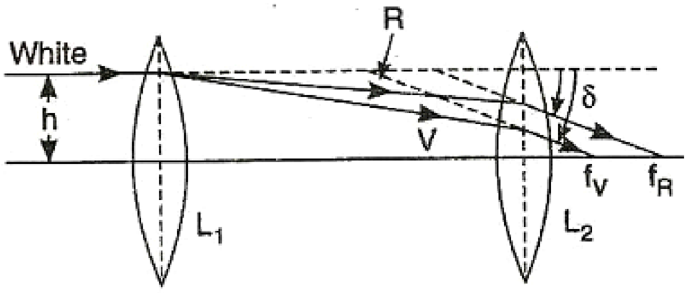

Fig. (24)

If F is the combined focal length of the two lenses then,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2} \rightarrow (1)$$

The change in $\frac{1}{F}$ can be determined by differentiating equation (1), we get

$$\therefore d\left(\frac{1}{F}\right) = d\left(\frac{1}{f_1}\right) + d\left(\frac{1}{f_2}\right) - d\left(\frac{x}{f_1 f_2}\right)$$

$$\Rightarrow d\left(\frac{1}{F}\right) = d\left(\frac{1}{f_1}\right) + d\left(\frac{1}{f_2}\right) - \frac{x}{f_2}\, d\left(\frac{1}{f_1}\right) - \frac{x}{f_1}\, d\left(\frac{1}{f_2}\right)$$

$$\Rightarrow d\left(\frac{1}{F}\right) = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - \frac{x}{f_2}\left(\frac{\omega_1}{f_1}\right) - \frac{x}{f_1}\left(\frac{\omega_2}{f_2}\right) \rightarrow (3)$$

$$\because d\left(\frac{1}{f_1}\right) = \frac{\omega_1}{f_1} \text{ and } d\left(\frac{1}{f_2}\right) = \frac{\omega_2}{f_2}$$

For an achromatic combination, $\frac{1}{F}$ should not change with colour.

That is, $d\left(\frac{1}{F}\right) = 0$ …(4)

From equations (3) and (4) we get,

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - \frac{x}{f_2}\left(\frac{\omega_1}{f_1}\right) - \frac{x}{f_1}\left[\frac{\omega_2}{f_2}\right] = 0$$

$$\Rightarrow \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = \frac{x(\omega_1 + \omega_2)}{f_1 f_2}$$

$$\Rightarrow \frac{\omega_1 f_2 + \omega_2 f_1}{f_1 f_2} = \frac{x(\omega_1 + \omega_2)}{f_1 f_2} \Rightarrow \omega_1 f_2 + \omega_2 f_1 = x\,(\omega_1 + \omega_2)$$

$$\Rightarrow x = \frac{\omega_1 f_2 + \omega_2 f_1}{\omega_1 + \omega_2} \rightarrow (5)$$

When the two lenses are made of same material then, $\omega_1 = \omega_2 = \omega$.

Hence, equation (5) can be written as,

$$x = \frac{\omega_1 f_2 + \omega_2 f_1}{\omega_1 + \omega_2} = \frac{\omega(f_2 + f_1)}{2\omega}$$

$$\Rightarrow \mathbf{x} = \frac{\mathbf{f_1 + f_2}}{\mathbf{2}}$$

Thus, in order to acquire the condition for achromatism, the separation between the two lenses must be equal to the average of the focal lengths of the two leases.

SOLVED PROBLEMS

1. *The focal lengths of thin convex lens are 100cm and 96.8cm for red and blue colours respectively. Find the dispersive power of the material of the lens. (S.V.U. 2016, March 2013, Sept 2011, March 2010)*

Sol.:

Focal length of convex lens for red colour, f_R = 100cm

Focal length of convex lens for blue colour f_b = 96.8cm

Dispersive power of the material of the lens,

$$\omega = \frac{f_R - f_B}{f}$$

Where, $$f = \frac{f_R - f_b}{2} = \frac{100 + 96.8}{2} = 98.4 \text{ cm}$$

$$\omega = \frac{100 - 96.8}{98.4} = \mathbf{0.0325}$$

2. *A convergent doublet of separated lens corrected for spherical aberration has an equivalent focal length of 10cm. The lens of the doublet are separated by 2cm. what are the focal lengths of its component lens. (S.V.U. March, 2012, March 2007, Sept 2007)*

Sol.:

Distance between the two lenses in the doublet, f_1-f_2 = x = 2cm $\Rightarrow$ f_1 = f_2+2

Where f_1 and f_2 are focal lengths of component lenses.

Equivalent focal lengths of doublet, F = 10cm.

Relation between f_1, f_2 and F is, $$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$$

$$\Rightarrow \frac{1}{10} = \frac{1}{(f_2 + 2)} + \frac{1}{f_2} - \frac{2}{f_2 (f_2 + 2)}$$

$$\Rightarrow \frac{1}{10} = \frac{f_2 + (f_2 + 2) - 2}{(f_2 + 2) f_2}$$

$$\Rightarrow f_2 = 18\text{cm} \quad \Rightarrow f_1 = f_2 + 2 = 20\text{cm}$$

$\therefore$ The focal lengths of component lenses are, **f_1 = 18cm** ; **f_2 = 20cm**

3. *The two thin lenses of focal lengths 16cm and 12cm form a combination which is corrected for spherical aberration. Find the distance between the lenses (S.V.U. Marc 2011)*

Sol.:

Focal length of the first lens, f_1 = 16cm

Focal length of the second lens f_2 = 12cm

For the combination of two lenses corrected for spherical aberration, the distance between the lenses is, $d = f_1 \sim f_2$

$\Rightarrow d = 16 \sim 12 = 4\text{cm}$

To correct the spherical aberration, the distance between the lenses is, **d = 4cm**

4. *A double convex lens has radii of curvature of 40cm and 10cm. Find the longitudinal chromatic aberration for object at infinity.*

Given $\mu_v = 1.523$; $\mu_r = 1.5\ 145$ (S.V.U. March 2009, S.V.U. March 2014)

Sol.:

Radius of curvature of first refracting surface of the lens, R_1 = 40cm

Radius of curvature of second refracting surface of the lens, R_2 = 10cm

Let f_R and f_v be the focal lengths of the convex lens for red and violet colours respectively.

The relation between f_R and μ_R is, $\frac{1}{f_R} = (\mu_R - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\Rightarrow \frac{1}{f_R} = (1.5145 - 1)\left(\frac{1}{40} - \frac{1}{10}\right) = \frac{0.5145 \times 3}{40} \Rightarrow f_R = \frac{40}{0.5145 \times 3} = 25.915\text{cm}$$

Similarly $\frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ $\Rightarrow \frac{1}{f_v} = (1.523 - 1)\left(\frac{1}{40} - \frac{1}{10}\right) = \frac{0.523 \times 3}{40}$

$$\Rightarrow f_v = \frac{40}{0.523 \times 3} = 25.493$$

For an object at infinity, the longitudinal chromatic aberration

$f_R - f_v = 25.915 - 25.493$ $\Rightarrow$ **$f_R - f_v = 0.422$cm**

5. *The objective glass of a telescope is an achromat of focal length 90cm. if the magnitude of the dispersive power of the two lenses are 0.024 and 0.036. calculate their focal lengths (S.V.U. Sept 2008)*

Sol.:

Combined focal length of achromat, F = 90cm

Focal length of first lens, f_1 = ?

Focal length of second lens f_2 = ?

Dispersive power of the first lens, ω_1 = 0.0.24

Dispersive power of the second lens, ω_2 = 0.036

For an achromatic doublet, $\frac{\omega_1}{f_1} = \frac{\omega_2}{f_2} = 0$

$$\Rightarrow \frac{0.024}{f_1} + \frac{0.036}{f_2} = 0$$

$\Rightarrow 0.024\ f_2 + 0.0.36\ f_1 = 0$ …(1)

The relation between f_1, f_2 and F is, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

$$\Rightarrow \frac{1}{90} = \frac{1}{f_1} + \frac{1}{f_2} \quad \ldots(2)$$

From equation (1) we have, $f_2 = \frac{-0.036}{0.024} f_1$

$\Rightarrow f_2 = 1.5\ f_1$ …(3)

From equations (2) and (3) we get,

$$\frac{1}{90} = \frac{1}{f_1} + \frac{1}{1.5} = \frac{1}{f_1}\left[1 + \frac{1}{1.5}\right] = \frac{1}{3f_1}$$

$$\Rightarrow f_1 = \frac{90}{3} = \Rightarrow f_1 = \mathbf{30cm}$$

$$\Rightarrow f_2 = -1.5 \times 30 = \mathbf{-45cm}$$

6. *Find the focal lengths of the two component lenses of an achromatic doublet of focal length 25cm. The dispersive powers of the crown and flint glasses are 0.022 and 0.044 respectively. (S.V.U. March 2008, March 2006)*

Sol.: Dispersive power of crown glass, ω_1 = 0.022

Dispersive power of flint glass ω_2 = 0.044

Dispersive of achromatic doublet, F = 25cm

Focal length of first lens, f_1 = ?

Focal length of second lens, f_2 = ?

Formula :- $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$

$$\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0$$

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}\Rightarrow\frac{1}{25}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$\Rightarrow\frac{1}{25}=\frac{f_1+f_2}{f_1\,f_2} \qquad \ldots(1)$$

$$\frac{0.022}{f_1}+\frac{0.044}{f_2}=0$$

$$\Rightarrow\frac{0.022}{f_1}=-\frac{0.044}{f_2}\Rightarrow f_2=-2f_1 \qquad \ldots(2)$$

From equations (1) and (2) we get,

$$\frac{1}{25}=\frac{f_1-2f_1}{f_1(-2f_1)}\Rightarrow\frac{1}{25}=\frac{-f_1}{-2f_1^2}=\frac{1}{2f_1}$$

$$\Rightarrow f_1=\frac{25}{2}=\mathbf{12.5\,cm}$$

$$f_2=-2f_1=\mathbf{-25cm}$$

7. *Calculate the focal length of a lens of dispersive power 0.031 which should be placed in contact with a convex lens of focal length 84 cm and dispersive power 0.021 to make the achromatic combinations. (O.U.2002)*

Sol.:

Dispersive power of first lens in achromatic doublet ω_1 = 0.021

Dispersive power of second lens; ω_2 = 0.031

Focal length of first lens f_1 = 84cm

Focal length of second lens, f_2 = ?

For an achromatic combinations,

$$\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0 \qquad \Rightarrow\frac{0.021}{84}+\frac{0.032}{f_2}=0$$

$$\Rightarrow\frac{0.021}{84}=-\frac{0.032}{f_2} \qquad \Rightarrow f_2=\frac{-0.032}{0.021}\times 84=-124\text{cm}.$$

Thus, the second lens is a concave lens of focal length **124cm.**

8. *It is desired to make a converging achromatic lens of mean focal length 30cm by using two lenses of materials A and B. If the dispersive powers of A and B are in the ratio 1:2 find the focal length of each lens. (S.V.U. 2002, S.K.U. 2001)*

Sol.:

Dispersive power of first lens A = ω_1

Dispersive power of second lens B = ω_2

From the data, $\omega_1 : \omega_2 = 1:2$

Focal length of combination of two lenses, F = 30cm

Focal length of first lens f_1 = ?

Focal length of second lens f_2 = ?

Formulae : $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \quad \Rightarrow \frac{-f_1}{f_2} = \frac{\omega_1}{\omega_2} = \frac{1}{2}$

$$\Rightarrow \; 2f_1 = f_2 \qquad \ldots(1)$$

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\Rightarrow \frac{1}{30} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \ldots(2)$$

From equations (1) and (2) we get,

$$\frac{1}{30} = \frac{1}{f_1} - \frac{1}{2f_1} = \frac{2f_1 - f_1}{2f_1^2} = \frac{f_1}{2f_1^2} \Rightarrow \frac{1}{30} = \frac{1}{2f_1} \Rightarrow f_1 = \frac{30}{2} = \mathbf{15cm}$$

f_2 = -2f = **- 30cm**

9. *A transparent sphere of radius 60cm is made up of a material of refractive index 1.5. Find the position of aplanatic points on the axis from the centre of the lens.*

Sol.:

Radius of transparent sphere, R = 60cm = 0.6m

Refractive index of material of the sphere, $\mu = 1.5$

The distance of aplanatic points from the centre of curvature are $\frac{R}{\mu}$ and $R\mu$.

$\therefore$ The distance of first aplanatic point, $d_1 = \frac{R}{\mu} = \frac{0.6}{1.5} \Rightarrow$ **d_1 = 0.4m**

The distance of second aplanatic point,

d_2 = $R\mu$ = 0.6 x 1.5 = 0.9,

10. A flint glass bio convex lens has radii of curvature +20cm and -20cm. If the refractive indices of flint glass for violet and red rays are given as 1.8 and 1.5. Find the longitudinal chromatic aberration.

Sol.:

Refractive index of flint glass of violet, $\mu_v = 1.8$

Refractive index of flint glass for red colour, $\mu_R = 1.5$

Radii of curvature of bio convex lens, $R_1 = +20cm$

$R_2 = -20cm$

Focal length of red rays, f_R is given by

$$\frac{1}{f_R} = (\mu_R - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5 - 1)\left[\frac{1}{20} - \frac{1}{(-20)}\right] \Rightarrow \frac{1}{f_R} = 0.5 \times \frac{2}{20}$$

$\Rightarrow f_R =$ **20cm**

Focal length of violet rays, f_v is given by

$$\frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.8 - 1)\left[\frac{1}{20} - \frac{1}{20}\right]$$

$$\Rightarrow \frac{1}{f_v} = 0.8 \times \frac{2}{20} \Rightarrow f_v = \frac{20}{2 \times 0.8} = \mathbf{12.5cm}$$

Longitudinal chromatic aberration is given by $f_R - f_v = 20 - 12.5 = \mathbf{7.5\,cm}$

IMPORTANT UNIVERSITY QUESTIONS

ESSAY TYPE QUESTIONS

1. What is meant by spherical aberration and chromatic aberration? Explain how chromatic aberration is eliminated by a combination of lenses in contact.
2. What is chromatic aberration? Derive the conditions for achromatism for two lenses when (i) placed in contact and (ii) separated by a distance. (2017)
3. Explain Coma and astigmatism. (2016, 2017)
4. What do you mean by curvature of the field and distortion? How they can be reduced? (2016)

SHORT ANSWER UESTIONS

1. Explain spherical aberration. (2016, 2017)
2. Write a note on aberrations. (M'11)
3. Write a short note on Astigmatism. (S'12, M'12)
4. Define and explain Coma. (S'11)

PROBLEMS TO BE SOLVED

1. *The focal lengths of convex lens are 50cm and 48cm for red and blue colours respectively. Find the dispersive power of the material of the lens.* ***(Ans.0.0408)***
2. *A Convergent doublet of separated lens corrected for spherical aberration has an equivalent focal length of 20 cm. The lens of the doublet are separated by 5cm. what are the focal lengths of its component lenses.* ***(Ans. 35cm,40cm)***
3. *Two thin lenses of focal lengths 20cm and 10cm form a combination which is corrected for spherical aberration. Find the distance between the lenses.* ***(Ans.10cm)***
4. *A double convex lens has radii of curvature of 50cm and 30cm. find the longitudinal chromatic aberration for object at infinity. (given μ_v=1.523; μ_R = 1.5145)* ***(Ans. 2.37cm)***
5. *Two thin convex lenses separated by a distance have equivalent focal length of 30cm. The combination satisfies the condition for the elimination of chromatic aberration and minimum spherical aberration. Find the values of the focal length of the two lenses and the distance between them.* ***(Ans.f_1=0.6m ; f_2 = 0.2m ; d = 0.4m)***
6. *A lens of dispersive power 0.0512 is kept in contact with a convex lens of focal length 20cm and dispersive power 0.0615. If the combination works as an achromatic doublet, find the focal length of the first lens.* ***(Ans : -16.65cm)***
7. *Two thin converging lenses of powers 2 diopters and 4 diopters are placed co-axially 20cm a part find the focal length of the combination.*

   (Ans: f_1 = 0.5m ; f_2 = 0.25m F =0. 0.227m)
8. *A convergent doublet of separated lenses, corrected for spherical aberration has an equivalent focal length of 25cm. The lenses in the combination are separated by 5cm. what are the focal lengths of the individual lenses* ***(Ans: f_1 = 0.5m ; f_2 = 0.45m)***
9. *In a telescope, the focal lengths of the objective and eye piece are 80cm and 20cm respectively what is the distance between the lenses to minimize chromatic aberration.* ***(Ans: 50cm)***
10. *Two lenses of focal lengths 40cm and 20cm are placed at a certain distance apart. Calculate the distance between them (i) to minimize chromatic aberration and (ii) to minimize spherical aberration. (**Ans: 30cm, 20cm)***